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A GENERALIZATION OF THE DRENICK–SHINOZUKA MODEL FOR BOUNDS ON THE SEISMIC RESPONSE OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM

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SUMMARY

This paper deals with seismic excitation of linearly elastic structures. It is well known that dynamical structural response is very sensitive to random fluctuations of the excitation; the consequence is that the identification of the forcing function on the basis of a few 'global' parameters does not allow to predict exhaustively stresses in seismically excited structures. In the paper, a convex model is established to treat uncertainty associated with the 'details' of the excitation, in order to set bounds on the response parameters of a SDOF system, assuming that only rough information is available for expected earthquakes at a given site. In particular, we assume that the maximum expected 'energy' of the accelerogram, and the maximum 'distance' of its power spectrum from the target spectrum, typical for the site under construction, are specified.

KEY WORDS: bounds; seismic; response

1. INTRODUCTION

Treatment of uncertainty is one of the most significant tasks for engineers, who are faced with the problem to design devices working under conditions that vary in a poorly predictable way. Traditionally, uncertainty was treated in a statistical context, by making use of the theory of probability and random processes. Later on, the fuzzy-set approach was developed to some extent, to model the uncertainty. Mostly, engineers deal with uncertainty through a deterministic transfer of the input into the output characteristics, obtained by probabilistic or possibilistic reasoning. Civil engineers, for instance, use some well-known procedures like the safety factors calibration or the limit state theory. In recent years, a proposal has been advanced in the monograph by Ben-Haim and Elishakoff,[1] to treat uncertainty through the mathematical theory of optimization, i.e. to find the worst combination of uncertain parameters. The ideas in Reference 1 are based on the realization of the fact that only well-established available data should be processed. A first formal setting of this approach was given in Reference 1 under the central assumption that uncertainty, i.e. the range of variation of uncertain items, is represented by a convex set, and that the characteristics of the performance of engineering systems could be modelled by a convex or non-convex function of the basic uncertain variables. If the performance characteristic is a convex function of basic uncertain variables, one can treat problems in the framework of convex analysis and obtain practical results through some established properties of convex

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optimization. One of the engineering areas where uncertainty is of utmost importance, is earthquake engineering. Uncertainty involves the time history of ground-shaking, since it, apart from some overall properties, appears to be quite unpredictable. Moreover, the structural response is highly sensitive to small changes and seemingly not significant details of the base excitation. The present paper aims at establishing basic criteria for treatment of uncertainty in seismic excitation through optimization, with the purpose to set up convex models for forecasting the maximum possible response parameters of a SDOF structure in the seismic environment.

2. PRELIMINARY CONSIDERATIONS

Consider the simple linear SDOF shear frame under the action of a ground acceleration $a(t)$. The equation of motion reads

$$\ddot{u} + 2\mu\Omega_0\dot{u} + \Omega_0^2 u = -a(t) \tag{1}$$

where $u(t)$ is the response, $a(t)$ the ground acceleration, Ω_0 the natural frequency of the undamped system and μ the damping coefficient.

The initial conditions read

$$u(0) = 0, \quad \dot{u}(0) = 0 \tag{2}$$

The solution, as well known, is given by

$$u(t) = -\int_0^t a(s)h(t-s)\,\mathrm{d}s \tag{3}$$

where $h(t)$ is the impulse response function

$$h(t) = \begin{cases} \dfrac{1}{\Omega_\mathrm{d}}\exp(-\mu\Omega_0 t)\sin(\Omega_\mathrm{d} t), & \text{if } t \geqslant 0 \\ 0, & \text{if } t < 0, \quad \Omega_\mathrm{d} = \Omega_0\sqrt{1-\mu^2} \end{cases} \tag{4}$$

The basic idea, first suggested by Drenick[2], is to find the base accelerogram function yielding the maximum possible value of the response $u(t)$. Following Drenick, assume that $a(t)$ has a finite duration, say T, and a bounded energy E_0,

$$\int_0^T a^2(t)\,\mathrm{d}t = \|a\|^2 = E_0^2 \tag{5}$$

Let $P(E_0)$ be the set of functions defined in interval $(0, T)$ and possessing the given energy E_0^2 then the problem reduces to

$$\begin{aligned} &\text{Maximize} \quad u(t) = \|u\|_T = E_0 H_T, \quad t \in (0, T) \\ &\text{subject to} \quad a \in P(E_0) \end{aligned} \tag{6}$$

As it was proved by Drenick

$$H_T^2 = \max_{t\in(0,T)} \int_0^T h^2(t-s)\,\mathrm{d}s = \int_0^T h^2(t_0 - s)\,\mathrm{d}s \tag{7}$$

As $T \to \infty$, the base accelerogram $a_\mathrm{c}(t)$ yielding the maximum peak response, and coined by Drenick in a later paper[3] as the *critical excitation*, has a shape coincident with the impulse response function reversed

with respect to time, and is given by

$$a_c(t) = \pm \frac{E_0}{H_\infty} h(-t) \tag{8}$$

The maximum peak response is given by

$$\| u \| = u_c(0) = E_0 H_\infty \tag{9}$$

where $u_c(t)$ denotes the response to be due to acceleration equal $a_c(t)$. Some criticism was expressed by Drenick[3] himself, mainly about the circumstance that aseismic design based on critical excitation could be "... far too pessimistic to be practical".

Yet Drenick investigated the ratio of the maximum response due to critical excitation, to the maximum response of actually recorded seismic accelerograms.[3] Response spectra were derived on the basis of both the critical excitation and a number of accelerograms from the set of recorded earthquakes, with attendant comparison of results. It was shown that for the range of structures with natural period from 0·5 to 1·2 s and damping coefficient in the range of 0·02–0·10, the ratio of the critical to be experimental response was almost invariably in the neighbourhood of two. By extending the class of possible accelerograms from only the recorded ones to linear combinations of these, the above ratio reduces to the range of values between 1·3 and 1·6. In later papers, Baratta[4,5] attempted to develop a modified procedure to obtain the least favourable excitation searching in the set produced by a generator of artificial earthquake accelerograms. The model chosen for such a generator followed the procedure of Ruiz and Penzien.[6] The class of admissible seismograms was constrained by a specified value of the maximum peak ground acceleration (PGA). The generator's parameters were calibrated on the basis of four available accelerograms recorded at Tolmezzo (Udine, Italy) during the earthquake of May 1976 (namely earthquakes of 6–9 May 1979), so that the class of admissible accelerograms was compatible with these samples. The comparison of the least-favourable responses of the *undamped* structure with the envelope of responses obtained through the processing of four recorded accelerograms, also yielded over-shooting values of about two in the range of structure's vibrational period of about 0·5 s, to the value of seven for more deformable structures.

The result appears to confirm that at least Drenick's critical response, if not the critical excitation, should be in the 'neighbourhood' of some realizable excitation during an earthquake, if the behaviour of the structure is in the elastic range. Analogous results were confirmed by Baratta and Zuccaro[7] by re-evaluation of the same seismic model. In this case, the analysis was carried on for 5 per cent damping, the overshooting ratio turns out to be around $2 \div 2{\cdot}5$, in a much wider range around $T_0 = 0{\cdot}5$ s. The extension of this range was probably due to the influence of damping. Again, Drenick's results may be well confirmed: *Earthquake-type functions of a given site may given responses that are comparable to those by critical excitation.*

It must be also stressed that, by introducing the least-favourable earthquake in a full probabilistic analysis of seismic hazard, the severity of the approach is quite mitigated by the combination with uncertainties deriving from regional seismicity.[7] It is possible that inelastic excursions of the structure can further reduce the gap between worst excitation and recorded one.[8]

From the above considerations, one can conclude that the original idea of the critical excitation is worthy of further pursuing. This is why this study is conducted, namely, to set up a new convex model to predict the worst possible shapes of accelerograms acting on linear structures, following the early idea by Shinozuka[9] that some bound on spectral ordinates can be used to improve *a priori* knowledge of the excitation. In this paper we modify and combine the approaches of Drenick[2] and Shinozuka.[9]

3. BASIC IDEA

We are interested in the maximum stress the structure can attain by earthquake action at a given site. Assume that the structural response does not exceed the appropriate linear threshold. Finally, assume that the base

accelerogram belongs to a linear space L, the set of *possible accelerograms*, whose basis is composed of accelerograms a_i ($i = 1, 2, \dots$), so that any possible accelerogram $a(t)$ can be expanded in the linear series of the base accelerograms:

$$a(t) = \sum_{i=1}^{\infty} c_i a_i(t) \tag{10}$$

Due to practical numerical reasons this series should be truncated to N, where N is the number of retained terms. One simple example of such a space is the Fourier expansion. As a result every accelerogram can be represented by a linear combination of harmonic functions.

Let us assume that the duration T of the earthquake is known, and that it is possible (from geophysics, for instance) to find a bound for the energy of the excitation, as in Drenick's problem

$$\int_0^T a^2(t)\,\mathrm{d}t \leqslant E_0^2 \tag{11}$$

If the base functions $a_i(t)$ are orthogonal and all scaled to the same energy, every function of the type (10) satisfying the constraint (11), will possess coefficients c_i satisfying the following inequality:

$$\sum_{i=1}^{N} c_i^2 \leqslant 1 \tag{12}$$

Any possible function that satisfies equation (11) is referred to as an *admissible accelerogram*. Thus, admissible functions can be mapped in the Euclidean space of N-dimensional vectors $\mathbf{c} = [c_1, c_2, \dots, c_N]$. Admissible accelerograms are contained into the unit sphere S_0, centred at the origin.

The values of the maximum and minimum displacements of the structure are required then for $a(t)$ varying within S_0. To this end, consider $u_i(t)$, the response function under $a_i(t)$. The response under any $a(t)$ expressed by equation (10), due to the system's linearity,

$$u(t) = \sum_{i=1}^{N} c_i u_i(t) \tag{13}$$

Thus, with matrix notation, denoting vectors and matrices by boldface characters and transposition by primes, the problem is set as follows for any fixed instant t in $(0, T)$:

$$\begin{gathered}\text{Find } \mathbf{c} \in R^N, \text{ for given } t\\ \text{such that } \mathbf{u}'(t)\mathbf{c} = \max,\ \mathbf{c}'\mathbf{c} \leqslant 1\end{gathered} \tag{14}$$

It is well known that in this case, a convex linear function to be optimized within a convex domain, the attendant solution point is on the boundary of S_0:

$$\begin{gathered}\text{Find } \mathbf{c} \in R^N, \text{ for given } t\\ \text{such that } \mathbf{u}'(t)\mathbf{c} = \max,\ \mathbf{c}'\mathbf{c} = 1\end{gathered} \tag{15}$$

which differs from equation (14) in that the inequality in equation (14) is replaced by an equality. Let us now introduce notations

$$r(\mathbf{c}) = \mathbf{u}'\mathbf{c}, \qquad f(\mathbf{c}) = \mathbf{c}'\mathbf{c} - 1 \tag{16}$$

At the solution point, with $k \geqslant 0$,

$$\mathbf{grad}\, f = k\, \mathbf{grad}\, r \quad \text{for } \mathbf{u}'\mathbf{c} = \max \tag{17}$$

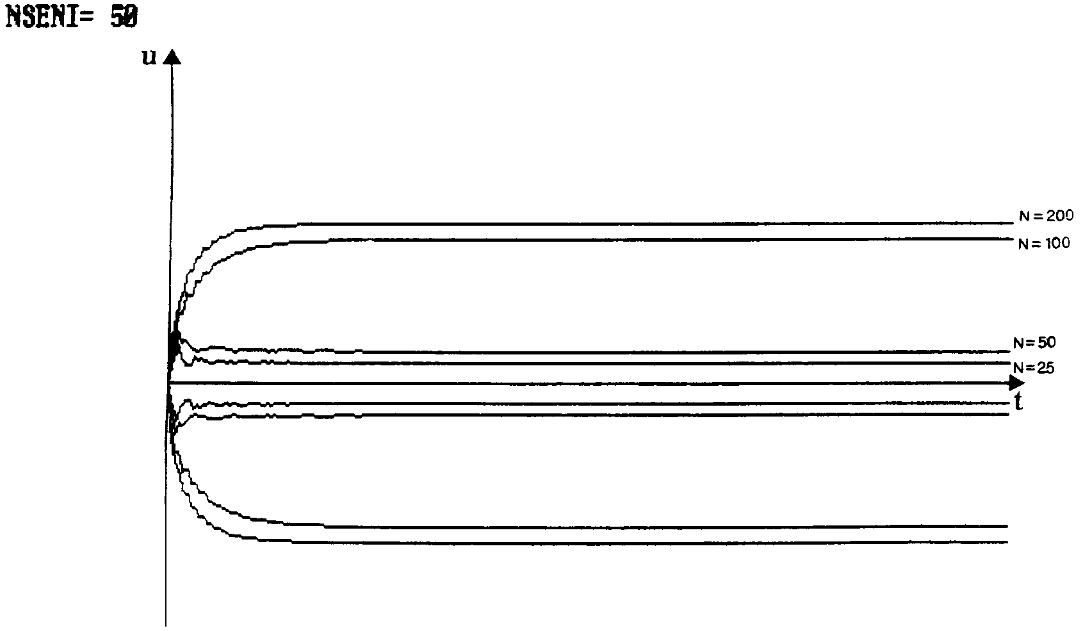

Figure 1. Influence of N, the number of harmonics, on the bound (21)

and

$$\mathbf{grad}\, f = -\, k\, \mathbf{grad}\, r \quad \text{for } \mathbf{u}'\mathbf{c} = \min \tag{18}$$

Hence

$$\mathbf{c} = \pm\, h(t)\mathbf{u}(t) \tag{19}$$

with

$$h^2(t) = 1/(\mathbf{u}'\mathbf{u}) \tag{20}$$

The maximum displacement at any instant t in (0, T), is finally given by

$$u_{\max}(t) = -\, u_{\min}(t) = \sqrt{\mathbf{u}'(t)\mathbf{u}(t)} \tag{21}$$

A numerical example has been carried by assuming that admissible functions are continuous. All a_i's were taken as harmonic functions, as in the Fourier expansion:

$$a_i(t) = \begin{cases} A_0 \sin(\phi_i t), & i \leqslant m \\ A_0 \cos(\phi_i t), & m < i \leqslant N \end{cases} \tag{22}$$

$$\phi_i = \frac{2\pi i}{T}, \qquad A_0^2 = \frac{2}{T} E_0^2$$

with N even, and $m = N/2$.

In Figure 1, the bound (21) is plotted for different values of $m = 25 \div 200$ for t ranging from $0 \div 4$ s. The figure shows how the refinement of the functional space of the excitation makes the bounds to increase progressively, and a good convergence is attained for $m = 100$. Figure 2 depicts a sample maximizing

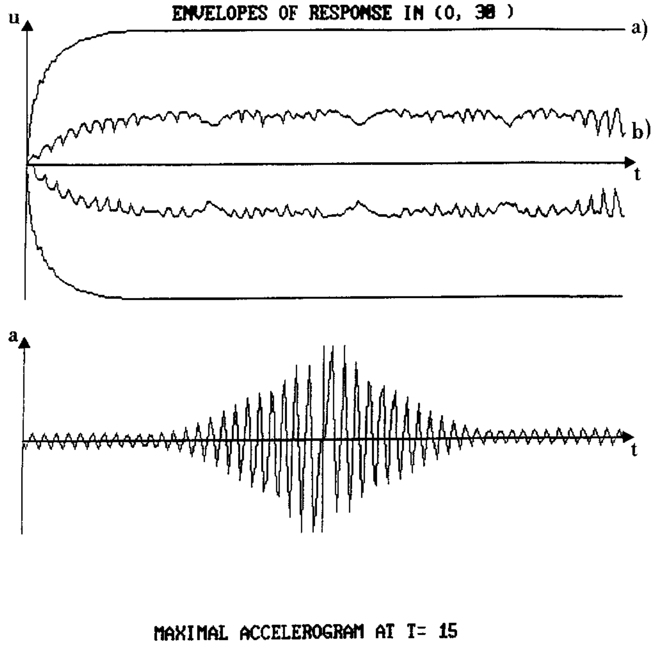

Figure 2. Upper and lower bounds and sample maximal accelerogram for $T=30$ s: (a) bounds (21); (b) uimax of any harmonic component

function for $T=30$ and $m=100$. These bounds are of the type of Drenick's critical excitation. It appears as highly questionable that such functions can be accepted as credible accelerograms.

4. CREDIBLE ACCELEROGRAMS

It is possible to investigate how the above bound is affected by presence of an *additional* information. Assume that earthquakes, in general, exhibit a nominal power spectrum whose shape is constructed to fit the following expression:

$$f(\phi\,|\,\sigma_1, \alpha_1, \sigma_2, \alpha_2) = [\,g(\phi\,|\,\sigma_1, \alpha_1) + g(\phi\,|\,\sigma_2, \alpha_2)]K \tag{23}$$

with

$$g(\phi\,|\,\sigma, \alpha) = \frac{\sigma^2 + 4\alpha^2\phi^2}{(\sigma^2 - \phi^2)^2 + 4\alpha^2\phi^2} \tag{24}$$

where σ, α are parameters governing the *shape* of the spectrum, and K is a normalizing factor. These parameters are assumed to have assigned values for any given site under examination. Note that equation (23) is nothing else but the sum of two functions of the Kanai–Tajimi type; the superposition is introduced in order to have a better approximation for spectra of recorded earthquakes exhibiting more than one dominant frequency.

Once this shape is assumed as the *central* spectrum, it is to be expected that accelerograms should not differ from the central spectrum by more than a given amount. Let $\mathbf{c}_0$ be the vector of combination coefficients fitting the central spectrum, i.e. with reference to a basis of the type (22):

$$c_{0i}^2 = c_{0,i+m}^2 = f(\phi_i\,|\,\sigma_1, \alpha_1, \sigma_2, \alpha_2) \quad (i \leqslant m) \tag{25}$$

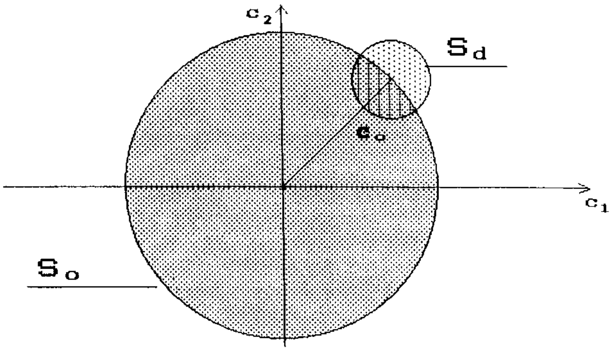

Figure 3. The set of admissible coefficients

We assume that through the analysis of the previous recorded earthquakes, it is possible to qualify earthquakes, and their variability, as accelerograms with given norm (equation (11)) as in the previous section, but with 'distance' from $\mathbf{c}_0$ not larger than a given amount, say θ. Thus, the constraint on the coefficients is of the type

$$\sum_{i=1}^{N} c_i^2 = 1 \tag{26}$$

$$\sum_{i=1}^{N} (c_i - c_{0i})^2 \leqslant \theta^2 \tag{27}$$

which should be considered in conjunction with the condition

$$\sum_{i=1}^{N} c_{0i}^2 = 1 \tag{28}$$

expressing the fact that $\mathbf{c}_0$ yields an accelerogram of a given norm, realized by assigning E_0 a proper value. The introduction of this additional information implies that one intends to deal with functions that are not 'far' from earthquake-type accelerograms.

The expected result is the maximum stress in a linear SDOF-structure acted-upon by spectrum-compatible earthquakes. The term *spectrum-compatible* indicates that at the given site the seismic excitation is qualified by its 'distance' (27) from the nominal central spectrum.

The problem is similar to the one dealt with in the previous section, except the fact that now the admissible set of excitations is given by the intersection of the unit sphere S_0 in equation (26), representing the 'energy' constraint, and the side-sphere S_d equation (27), namely the spectrum-compatible sphere (Figure 3). The problem reads as follows:

$$\text{Find } \mathbf{c} \in R^n\text{: such that } \mathbf{u}'(t)\mathbf{c} = \max, \mathbf{c}'\mathbf{c} = 1 \tag{29}$$
$$(\mathbf{c} - \mathbf{c}_0)'(\mathbf{c} - \mathbf{c}_0) \leqslant \theta^2$$

where $\theta^2 \leqslant 4$, otherwise S_d includes S_0, and the problem reduces to the formulation given in equation (15). Problem (29) can be solved in two steps:

Step 1: Solve the simpler problem (15) and find $\mathbf{c}_1$. If $\mathbf{c}_1$ is contained in S_d, then it represents also the solution of the problem (29). If $\mathbf{c}_1$ is on S_0 but is outside S_d then no internal point to S_d exists where the optimal condition (17) [respectively, equation (18)] holds, and the solution must be searched on the boundary of the intersection of S_0 and S_d, as described in step 2.

Step 2: Now the problem transforms into the following one:

$$\text{Find } \mathbf{c} \in R^n\text{: such that } \mathbf{u}'(t)\mathbf{c} = \max, \ \mathbf{c}'\mathbf{c} = 1$$
$$(\mathbf{c} - \mathbf{c}_0)'(\mathbf{c} - \mathbf{c}_0) = \theta^2 \tag{30}$$

Note that the constraint concerning S_d can be written

$$\sum_{i=1}^{N} (c_i - c_{0i})^2 = \sum_{i=1}^{N} (c_i^2 - 2c_i c_{0i} + c_{0i}^2) = \theta^2 \tag{31}$$

that, taking into account equation (28), we can state that

$$\sum_{i=1}^{N} (c_{0i} c_i) = 1 - \theta^2/2 = a \tag{32}$$

with

$$|a| \leqslant 1 \tag{33}$$

As a consequence, the problem can be formulated as follows:

$$\text{Find } \mathbf{c} \in R^n\text{: such that } \mathbf{u}'(t)\mathbf{c} = \max, \ \mathbf{c}'\mathbf{c} = 1, \ \mathbf{c}_0'\mathbf{c} = a \tag{34}$$

where all constraints appear in the form of equalities. In order to solve the latter problem, we consider the Lagrangian function

$$L(\mathbf{c}, r_1, r_2 \,|\, t) = \mathbf{u}'(t)\mathbf{c} + r_1(\mathbf{c}'\mathbf{c} - 1) + r_2(\mathbf{c}_0'\mathbf{c} - a) \tag{35}$$

where r_1 and r_2 are Lagrange multipliers. We require the gradient of the Lagrangian with respect to $\mathbf{c}$ vanish

$$\mathbf{grad}_\mathrm{c}\, L = \mathbf{u}(t) + 2r_1\mathbf{c} + r_2\mathbf{c}_0 = \mathbf{0} \tag{36}$$

Hence,

$$\mathbf{c} = \mathbf{c}(t) = -\frac{1}{2r_1}\,[r_2\mathbf{c}_0 + \mathbf{u}(t)] \tag{37}$$

Substituting equation (37) into equation (26) for S_0 we obtain

$$\mathbf{c}'\mathbf{c} = \frac{1}{4r_1^2}\,(\mathbf{u}'\mathbf{u} + r_2^2\mathbf{c}_0'\mathbf{c}_0 + 2r_2\mathbf{u}'\mathbf{c}_0) = 1 \tag{38}$$

where one puts

$$\mathbf{u}'(t)\mathbf{c}_0 = b, \quad \mathbf{u}'(t)\mathbf{u}(t) = 4U^2(t) \tag{39}$$

Bearing in mind that for the solution we must have, $\mathbf{c}'\mathbf{c} = \mathbf{c}_0'\mathbf{c}_0 = 1$, we arrive at

$$4r_1^2 - r_2^2 - 2r_2 b - 4U^2(t) = 0 \tag{40}$$

Substituting now equation (37) into equation (27) for S_d with equality sign we get

$$\mathbf{c}_0'\mathbf{c} = -\frac{1}{2r_1}\,(r_2 + b) = a \tag{41}$$

Hence

$$r_2 = -(2r_1 a + b) \tag{42}$$

Substituting the latter expression into equation (40) the following equation for r_1 is obtained:

$$4(1-a^2)r_1^2 + b^2 - 4U^2 = 0 \tag{43}$$

Therefore

$$r_1 = \pm\frac{1}{2}\sqrt{\frac{4U^2 - b^2}{1-a^2}} \tag{44}$$

Note that the expression under the root is positive. Indeed, by elementary use of the Cauchy–Schwartz inequality we establish

$$b^2 = [\mathbf{c}'_0\mathbf{u}(t)]^2 \leqslant [\mathbf{c}'_0\mathbf{c}_0]\,[\mathbf{u}'(t)\,\mathbf{u}(t)] = \mathbf{u}'(t)\mathbf{u}(t) = 4U^2 \tag{45}$$

After r_1 has been calculated, r_2 is given by equation (42). Consider now the elements of the Hessian

$$\frac{\delta^2 L(c, r_1, r_2 \mid t)}{\mathrm{d}c_i \mathrm{d}c_j} = \begin{cases} 2r_1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \tag{46}$$

Thus, we verify that the matrix of second derivatives of the Lagrangian with respect to the basic variables c_i is diagonal. Moreover, it is positive-definite if r_1 is positive; then the solution corresponds to the conditional minimum of the objective function $\mathbf{u}'\mathbf{c}$. Analogously, if $r_i < 0$, the matrix of second derivatives of the Lagrangian is negative-definite, and the solution corresponds to relative maximum of the objective function.

In conclusion, the solution for the problem of the *maximum* response is given by

$$\begin{aligned} r_1 &= -\frac{1}{2}\sqrt{\frac{4U^2 - b^2}{1-a^2}} \\ r_2 &= -(2r_1 a + b) \\ \mathbf{c} = \mathbf{c}(t) &= -\frac{1}{2r_1}[r_2\mathbf{c}_0 + \mathbf{u}(t)] \end{aligned} \tag{47}$$

Likewise, the solution for the *minimum* response is given by changing the sign of r_1:

$$\begin{aligned} r_1 &= -\frac{1}{2}\sqrt{\frac{4U^2 - b^2}{1-a^2}} \\ r_2 &= -(2r_1 a + b) \\ \mathbf{c} = \mathbf{c}(t) &= -\frac{1}{2r_1}[r_2\mathbf{c}_0 + \mathbf{u}(t)] \end{aligned} \tag{48}$$

5. APPLICATION

In order to have a quantitative estimate of the proposed procedure, consider a particular area, namely the Campania region in Southern Italy, where a number of accelerograms have been recorded during the Campano-Lucano earthquake which took place 23 November 1980. The magnitude of the event was estimated to be 6·5, the epicentral intensity was set at the 7·5° of the MCS scale. The event was rather non-typical, mainly because of its duration that lasted up to almost 2 min in some sites. It should be noted that, only the duration of the strong-phase motion will be considered here, that varies from about 52 to more than 86 s. Information on the local characters of ground motion was derived from the direct inspection of the recorded accelerograms, limiting the analysis, for simplicity, to only the NS component. The considered records are summarized in Table I. Recorded ordinates of all accelerograms are converted to $\mathrm{cm\,s^{-2}}$. All earthquakes are preliminarily reduced to the same energy norm, given in equation (5) as the one recorded in

Table I

Site	Epicentral distance (km)	Local intensity (MSK)	PGA (g/10)	Duration (s)	Norm (cm $s^{-3/2}$)
Torre del Greco	80·1	7·0	0·59	52·9	79·46
Brienza	41·3	7·0	2·24	78·7	185·19
Sturno	34·8	6·0	2·25	70·7	284·53
Calitri	21·0	8·5	1·52	86·1	268·25
Bagnoli	22·3	6·0	1·31	79·1	152·29

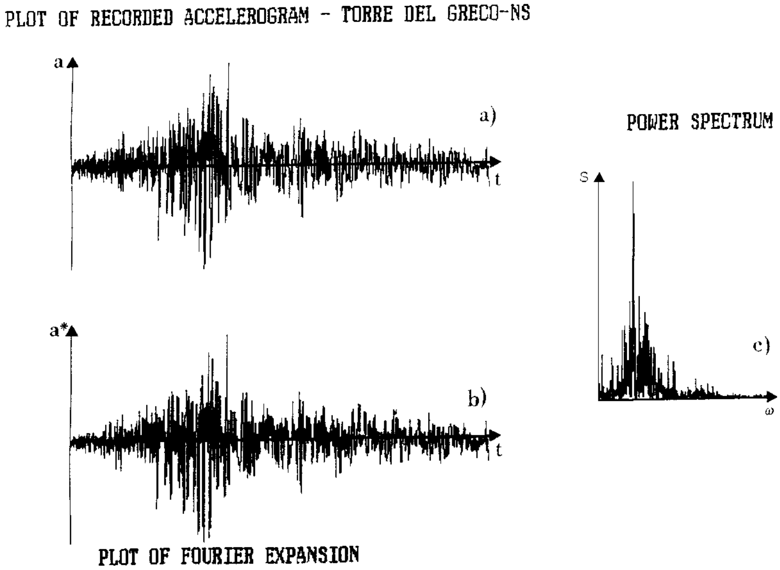

Figure 4. TORRE DEL GRECO–Campania Earthquake of 11/23/80; (a) recorded accelerogram; (b) Fourier approximation; (c) power spectrum

Torre del Greco, the closest site to Naples, the largest town in the region, so that every record possesses energy $E_0 = 79{\cdot}46\ \mathrm{cm\,s^{-3/2}}$.

The analysis was carried on up to include $m = 400$ sine and cosine waves for Torre del Greco, Sturno and Calitri, while this number is increased up to $m = 800$ waves for the accelerograms recorded in Brienza and Bagnoli Irpino, that exhibit power spectra scattered on a wider range of frequencies. A plot of a typical accelerogram, with the approximation resulting from the associated Fourier expansion and its power spectrum is given in Figure 4.

Let f_{ij} be the ith coefficient of the Fourier expansion of the jth accelerogram ($i = 1, \dots, m; j = 1, \dots, 5$). The central distribution of coefficients, collected in the vector $\mathbf{c}_0$, were obtained for each of the five analysed sites, through the following steps:

(1) Normalization of f_{ij}, for every j, to unit modulus through division by a factor $F_j = |\mathbf{f}_j|$, and letting $c_{ij} = f_{ij}/F_j$.

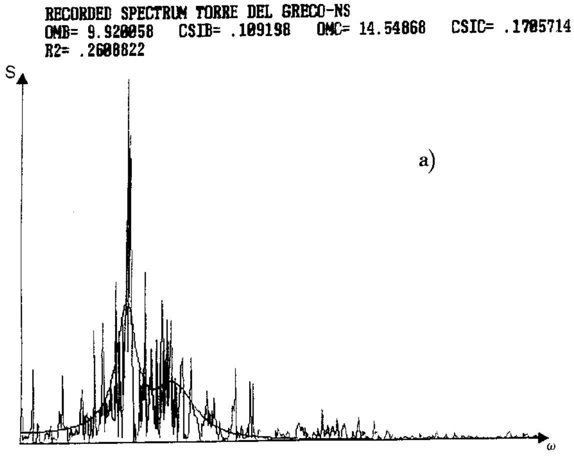

Figure 5. Processed spectra from recorded accelerograms and fitted central spectra, in the location of Torre del Greco

(2) Determination for every site, of the best values of the parameters σ_j, α_j, that give the optimal fit of the ordinates of the Kanai–Tajimi-type spectrum given in equation (23) with the values of the spectrum corresponding to the normalized coefficients c_{ij} from the recorded accelerograms on the corresponding pulsations.
(3) Calculation for every site of the ordinates s_{0ij} of the target spectrum, as follows:

$$s_{0ij}^2 = f(\phi_i | \sigma_1, \alpha_1, \sigma_2, \alpha_2), \qquad \sum_{i=1}^{m} s_{0ij}^2 = 1 \tag{49}$$

(4) Letting

$$\beta_{ij} = \frac{c_{i+m,j}}{c_{ij}} \tag{50}$$

and, finally, calculating the ordinates of the nominal target spectra at the considered sites as follows:

$$\begin{aligned} c_{0ij} &= \frac{s_{0ij}}{\sqrt{1+\beta_i^2}} \operatorname{sgn}(f_{ij}) \\ c_{0,i+m,j} &= \beta_{ij} c_{0ij} \quad (i = 1, \dots, m) \end{aligned} \tag{51}$$

(5) Calculation for every site of the quadratic scatter of the coefficients obtained by directly processing the accelerogram and those calculated by the target spectrum

$$\theta_j^2 = \sum_{1=1}^{m} (c_{ij} - c_{0ij})^2 \tag{52}$$

In Figure 5 the processed spectra for the considered sites are plotted and superimposed to the corresponding central ones. Note that Figure 5 shows only the spectra in the location Torre del Greco. In Table II the final values of θ_j^2 are listed illustrating that the inequality $\theta^2 \leqslant 4$ is holding.

Assuming that the central spectrum is site-dependent, but that independence holds for the quadratic scatter, it is possible to infer from the data that for the area under examination $\theta^2 \approx 0{\cdot}26$. After introducing this value in the results discussed in Section 4, one gets the bounds plotted in Figures 6 and 7, for

Table II

Site	θ^2	σ_1 (s^{-1})	α_1	σ_2 (s^{-1})	α_2
Torre del Greco	0·261	9·92	0·109	14·55	0·171
Brienza	0·219	32·98	0·478	37·67	0·403
Sturno	0·186	17·94	0·347	3·77	0·399
Calitri	0·358	0·07	0·059	6·35	0·211
Bagnoli Irpino	0·257	5·15	0·142	28·64	0·762

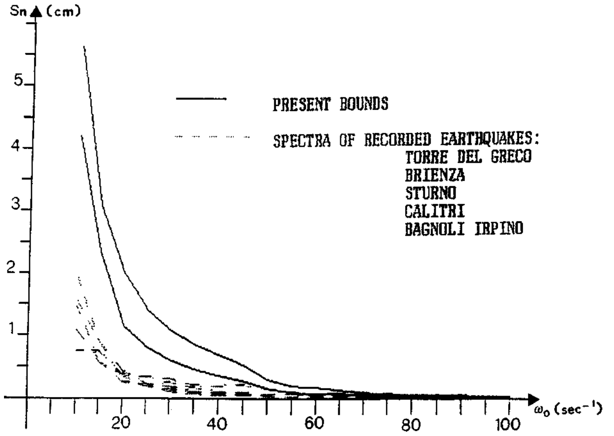

Figure 6. Response spectra of displacement: Comparison of present bounds with the envelope of spectra from recorded accelerograms normalized to the same norm as Torre del Greco

$\Omega_0 = 30 \div 60\,s^{-1}$. It is also possible to perform a comparison with the previous, spectrum-free bound, showing that the new bound is approximately half of the one constrained only by the upper bound by Drenick.

The response displacement spectrum calculated by the present procedure is finally calculated, for a value of the damping coefficient $\mu = 0{\cdot}05$. The comparison with the envelope of the same spectra calculated with reference to the accelerograms processed, shows that the present results, although better founded and less sensitive to uncertain parameters than recorded accelerograms, are not too conservative, but yield a magnification by a factor of about two in the design forces, in the whole range of natural periods of the structures; this result can be directly incorporated into the safety factor.

The same spectra obtained via the procedures developed in Sections 3 and 4, were numerically compared with the response spectrum yielded by Drenick's approach[2] and the present one. As expected, Drenick's bound is practically coincident with the results yielded by problem in equation (15).

Following Shinozuka's [10] model, and the assumption given in equation (23), with the parameters presented in Table II, one obtains the following bound for the maximum displacement over the whole duration of the excitation:

$$I_{ei} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |H(\phi)|\,X(\phi)\,d\phi \tag{53}$$

where

$$H^2(\phi) = \frac{1}{(\Omega_0^2 - \phi^2)^2 + 4\mu^2\Omega_0^2\phi^2} \tag{54}$$

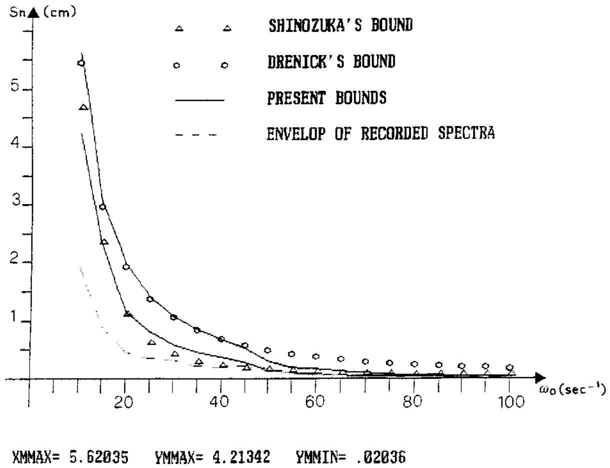

Figure 7. Response spectra of displacement: Comparison of present bounds with Drenick's and Shinozuka's spectra

and

$$X(\phi) = K[\,g(\phi\,|\,\sigma_1, \alpha_1) + g(\phi\,|\,\sigma_2, \alpha_2)] \tag{55}$$

with K chosen so that

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X^2(\phi)\,\mathrm{d}\phi = E_0^2 \tag{56}$$

From inspection of Figures 6 and 7 it is possible to infer that problem (15), the bound in the *norm*, yields results substantially coincident with Drenick's original approach, apart from the range of higher frequencies, due to the truncation of the expansion (22) by a finite number of harmonics (say, for $\phi_i > 50\ \mathrm{s}^{-1}$). Moreover, problem (29), the *spectrum-compatible* bound, is not significantly different from Shinozuka's approach, although in the range of low frequencies it yields slightly closer bounds. The opposite happens in the range of higher frequencies.

The ratio of the ordinates of the spectrum-compatible bound to the corresponding ordinates of the envelope of spectra related to actual seismic records, normalized to the same norm as Torre del Greco, very in the range $2{\cdot}5 \div 2{\cdot}0$. It is remarkable that this ratio decreases with θ^2, a new feature which was not exhibited by previous approaches.

6. DISCUSSION AND CONCLUSION

In this study a new approach to analyse the response of a linear structure subjected to the seismic loads and to calculate an upper bound of the response has been proposed, based on the solution of a convex optimization problem. Starting from a specialized function space, with a given basis, the optimal combination of the coefficients can be found analytically. The data required for the present analysis are: (i) the estimated energy of the earthquake; (ii) an estimate of the shape of the central power spectrum at the site; (iii) an upper bound on the maximum 'distance' of the power spectrum of realizable earthquakes at the site from the central one.

A direct comparison has been performed with Shinozuka's[10] approach, yielding at some cases very close results. It should be noted that in Shinozuka's approach, however, the admissible spectra are bounded from above by the 'central' spectrum, while the present approach includes earthquakes whose spectra are allowed

to exceed the central spectrum; note, moreover, that in Shinozuka's approach the bound involves only the *steady-state* part of the response, while in the present approach, as in Drenick's[2] one, the bound involves also the transient response starting with homogeneous initial conditions. The new convex modelling includes positive features of the approaches developed by Drenick[2] and Shinozuka,[10] and considerably reduces the estimates of the maximum possible response.

One should stress that the real structures are generally not SDOF-systems where the maximum stresses are governed by the maximum displacement of a single mode. The generalization of this method to multi-degree-of-freedom realistic structures is possible through application of the general methodology outlined by Ben-Haim and Elishakoff,[1] or alternatively, casting the multi-degree-of-freedom system in a state-space form as a vector differential equation. One should also note that the maximum stress at two distinct positions of the structure will be produced by the *different* worst-case combinations of excitations. Obviously, one is interested in the worst response at the critical location of the structure. Determining such a location may be a non-trivial task. Then, if the structure is represented as the multi-degree-of-freedom system, one must explore the maximum responses of each of the masses, and the maximum of these maxima will constitute a critical response. Analogously, a distributed system must be discretized through a fine mesh and the maximum responses at each nodal point must be determined. Then the location with maximum response amongst critical responses at each node will constitute the globally critical response. The attractive feature of this method is the fact that one should not 'guess' the critical location, but rather determine it through a discretization scheme combined with convex optimization procedure for each nodal point of the mesh.

The present approach is restricted to linear systems. The generalization to realistic non-linear systems appears to be necessary for practical applicability of this method. In this context, convex models have been applied to the non-linear structures through treating the non-linear transfer function between the uncertain quantities and the output of the non-linear transformation, as a numerical code in Reference 8 and automatic differentiation procedure. Another alternative is the use of the non-linear programming schemes. For the application of this method readers may consult Reference 11.

To sum up, it appears that the time is ripe to critically revisit existing stochastic approaches in the earthquake engineering and examine the possibility of utilizing alternative approaches. One such an alternative approach was attempted in this study, although to a SDOF system in the linear setting. Related subjects, although in different settings were pursued recently by Pantelides and Tzan[12,13] and Ben-Maim *et al.*[14] Non-probabilistic, convex modelling is both conceptually sound, and computationally less expensive than purely probabilistic method.

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